Edge Charge Singularity of Conductors

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In this paper, we investigate a diffusion-based simulation method for the edge singularity of a conducting object, rapidly evaluating the power-law singularity associated with the edge of the conducting object for the computation of capacitance coefficients for VLSI interconnection systems. We show that for the edge singularity of a conducting object the edge distribution in the last-passage method can be approximated to be proportional to the potential difference at a point very near to the conducting object from the potential of the conducting object. One exemplary application of this method to the cube edge is in good agreement with the previous result.

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I. INTRODUCTION

Numerical methods for the parasitic capacitance extraction in VLSI interconnects can be generally classified into two categories: charge integration methods and energy calculation ones [1]. In charge-based deterministic methods such as finite difference methods or finite element methods for the capacitance coefficients for VLSI interconnection systems, one of the major numerical problems arises from the charge singularities at the corners and edges of the interconnection systems. In addition to the previous corner singularity [2] exponent calculation method based on a first-passage diffusion simulation [3–5], in this paper we investigate the edge singularity of a conducting object based on the similar first-passage diffusion simulation. We expect to significantly contribute to the computation of capacitance coefficients for VLSI interconnects due to the rapid evaluation of the singularities.

II. THEORY

Close enough to the edge singularity, the surface charge density on a charged conductor will be dominated by a single, power-law singularity of the form [2,6–9]

\[ \sigma(\delta, x) = \delta^{\pi/\alpha - 1} \sigma_e(x) = A \cdot \delta^{(\pi/\alpha - 1) - \gamma}. \]

(1)

Here, \( \delta \) is the distance from the edge, \( \sigma_e \) the edge distribution, \( x \) the distance from the corner, \( A \) a constant and \( \alpha \) the angle between the two intersecting surfaces which form the edge (see Fig 1).

In the last-passage edge distribution method [6,7,9], the edge distribution can be given as:

\[ \sigma_e(x) = \frac{1}{4\pi} \lim_{\delta \to 0} \delta^{1-\pi/\alpha} \int_{y \in \partial \Omega_y} d^2 y G(x, y) p(y, \infty). \]

(2)

Here, \( \partial \Omega_y \) is an enclosing surface that intersects the pair of absorbing surfaces meeting at angle \( \alpha \) (as an example, see Fig 2 [6,9]). \( G(x, y) \) is the last-passage Green’s function and \( p(y, \infty) \) is the probability associated with a diffusing particle initiating at the point \( y \) on the enclosing first-passage (FP) surface and diffusing to infinity without ever returning to the conducting object.
The edge distribution has a natural probabilistic interpretation \cite{9} : It is the (rescaled) probability that a diffusing particle makes its last passage at the edge point \( x \). In this paper, we show that the edge distribution can be approximated as
\[
\sigma_e(x) = A[\Phi_0 - \Phi(x + \delta r)],
\]
where \( \Phi_0 \) is the potential of the conductor, \( \delta r \) the small distance from the edge, and \( A \) a constant.

### III. METHOD AND DISCUSSION

By the basic isomorphism of probabilistic potential theory \cite{9–11}, the potential at a point external to a conducting object can be equal to the probability of a diffusing particle that starts at the point and contacts the conductor. In our computations, the potential of the conductor \( \Phi_0 \) is set to be unity. We use “Walk on Planes” (WOP) \cite{3,12}, one of the Green’s function FP methods \cite{9,13,14}, to obtain the potential at a distance from the singularity.

In this paper, the method is illustrated by computing the edge distribution of the unit cube. We calculate the potential \( \Phi(x + \delta r) \) by performing simulations as follows \cite{12} : A Brownian walker is initially placed at \( x + \delta r \). After that, the Brownian walk is constructed as a series of FP jumps, each from the present location of the Brownian walker to a new location on the FP plane which includes one of the six faces of the unit cube (see Fig 3 \cite{12}). When the Brownian walker makes its FP jump at the distance \( d \) from the plane, we sample the radial FP point \( r \) on the FP plane by using the following:
\[
r \cdot d = \sqrt{(1 - \eta^2) / \eta^2}, \quad \eta \in [0, 1].
\]
Here, \( \eta \) is a random number (see Fig 4 \cite{12}). If the Brownian walker is placed inside the launching sphere but not on the face of the unit cube, another FP plane is chosen, and the Brownian walk continues. When the Brownian walker is placed outside the launching sphere, we decide whether the Brownian walker goes back to the launching sphere by using the probability \( P_{\text{inf}} \) of going to infinity \cite{15} without going back to the launching sphere,
\[
P_{\text{inf}} = 1 - b / r.
\]
Here, \( b \) is the launching sphere radius and \( r \) the radial position of the Brownian walker from the center of the launching sphere.

When it is determined to go back to the launching sphere, we use the replacement distribution density function \cite{15},
\[
\omega(\theta, \phi) = \frac{1 - \alpha^2}{4\pi[1 - 2\alpha \cos \theta + \alpha^2]^{3/2}}.
\]
Here, \((\theta, \phi)\) is defined with respect to the polar axis that joins the position of the Brownian walker to the launching sphere center, and \( \alpha = b/r \) (see Fig 5 \cite{12}). This procedure is repeated until the Brownian walker either reaches the conducting object or goes to infinity. After simulating \( N \) Brownian walks, we obtain the probability,
Fig. 5. This illustration shows the usage of the replacement distribution density function \( \omega(\theta, \phi) \) when the WOP jump from the launching sphere puts a random walker outside the launching sphere, and the probability \( P_{inf} \) decides that the Brownian walker goes back to the launching sphere.

Fig. 6. Edge distribution of the unit cube by using potential-based approximation (stars) compared with edge distribution method (circles). The edge distribution is shown rescaled with the value at the center of the edge of the cube. Here, \( x \) is the distance from the center of the edge of the cube.

\[
\Phi(x + \delta r) = \frac{N_\alpha}{N},
\]

where \( N_\alpha \) is the number of times of making contact on the object.

The result shown in Fig 6 is in good agreement with the previous result [6]. We should note that the method is not limited to the geometry of the cube and that the edge distribution needs to be calculated only once for each geometrically distinct edge.

IV. CONCLUSION

Our method combined with the previous corner singularity [2] can significantly contribute to the computation of capacitance coefficients for VLSI interconnects, due to the rapid evaluation of the singularities. Domain decomposition difficulty near corners and edges in charge integration deterministic methods for the capacitance coefficients of the VLSI interconnects can be overcome by the combined Monte Carlo methods: the corner singularity method near the corner, the edge singularity method far enough from the corner to approximate the charge singularity by the conventional infinite edge approximation, and the method of this paper for the intermediate edge singularity.

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